

The planetary spin and rotation period: A modern approach

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Abstract –Using a new approach, we have obtained a formula for calculating the rotation period and radius of planets. In the ordinary gravitomagnetism the gravitational spin (S) orbit (L) coupling, $\vec{L} \cdot \vec{S} \propto L^2$, while our model predicts that $\vec{L} \cdot \vec{S} \propto \frac{m}{M} L^2$, where M and m are the central and orbiting masses, respectively. Hence, planets during their evolution exchange L and S until they reach a final stability at which $MS \propto mL$. Rotational properties of our planetary system, binary pulsars and exoplanets are in agreement with our predictions. The radius (R) and rotational period (D) of tidally locked planet at a distance a from its star, are related by, $D^2 \propto \sqrt{\frac{M}{m^3}} R^3$ and that $R \propto \sqrt{\frac{m}{M}} a$.

introduction. – Kepler’s laws best describe the dynamics of our planetary system as regards to the orbital motion. However, Newton’s law of gravitation provided the theoretical framework of these laws. In central potential the orbital angular momentum is conserved. In polar coordinates, the gravitational force consists of the ordinary attraction gravitational force and a repulsive centripetal force. The Newton’s law of gravitation has been successful in many respect. However, this law fails to account for very minute gradational effect like deflection of light by an intervening star, precession of the perihelion of the planetary orbit and the gravitational red-shift of light passing a differential gravitational potential. Einstein’s general theory of gravitation generalizes Newton’s theory of gravitational to give a full account for all these observed gravitational phenomena. Einstein treats these phenomena as arising from the curvature of space. Hence, Einstein’s theory has become now the only accepted theory of gravitation. The inclusion of energy and momentum of matter (mass) in question leads to the curvature of space, while the inclusion of spin leads to torsion in space.

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Einstein's theory deals with matter of the former case, while Einstein-Cartan deals with the latter case. Thus, Einstein space is torsion free. In classical electrodynamics the spin of a particle is a quantum effect with no classical analogue. However, the spin of a gravitating object (e.g., planets) is defined as a rotation of an object relative to its center of mass. This is expressed as $S = I\omega$, where I and ω are the moment of inertia and angular velocity of the rotating object. The spin is generally a conserved quantity in physics. Besides the spin, an object (m) revolving at a distant r around a central mass (M) with speed v is described by its orbital angular momentum. This is defined as $L = \vec{r} \times m\vec{v}$. This quantity is also conserved, except when an external torque is acting on the object. In quantum mechanics the spin and angular momentum of a fundamental particle are quantized. No such quantization is deemed to exist in gravitation. To incorporate quantum mechanics in gravitation we invoke a Planck-like constant characterizing every gravitational system [6, 7]. This would facilitate a bridging to quantum gravity that has not yet been uniquely formulated so far.

The spin and orbital angular momentum may couple to each other as the case in the Earth-Moon system. Therefore, neither the spin nor the orbital angular momentum are separately conserved. Their sum is always conserved. A similar coupling occurs in atomic system. For instance, because of the spin of the electron an existence of such effect is found to be present in hydrogen-like atoms.

Owing to the existing similarities between gravitation and electromagnetism, some analogies were drawn which led to the gravitomagnetism paradigm. It is believed that an effect occurring in electromagnetism will have its counter analogue in gravitomagnetism.

In this paper we formulate the proper spin-orbit coupling in a gravitational system, and then deduce a formula for the spin of a gravitating object. This is done by equating the spin-orbit coupling energy to the gravitomagnetic energy. The resulting equation relates the spin of a gravitating object to its orbital angular momentum. While in standard gravitomagnetism, the gravitational spin-orbit coupling, $\vec{L} \cdot \vec{S} \propto L^2$, while in our model of gravitomagnetism predicts that $\vec{L} \cdot \vec{S} \propto \frac{m}{M} L^2$. This relation suggests a balance equation, $mL \sim MS$. For this reason any orbiting object must spin to be dynamically stable. So planets during their course of evolution exchange L and S , but eventually come to a state of stability. The larger the planet the larger its spin. Hence, Jupiter spins faster than other planets in the solar system. Equivalently, the spin $S \propto \frac{Gm^2}{v}$, where G is the gravitational constant and v is the orbital velocity. This formula is found to be consistent when applied to our planetary system, exoplanetary system, and binary pulsars. Astronomers have discovered so far more than 800 new giant planets, but couldn't identify all of their radii and spin periods. The present formulation helps identify these latter properties. We consider here all possibilities to account for the observational derived data for exoplanetary system and their consistency.

The gravitational spin-orbit coupling. – The spin - orbit interaction resulting from an interaction of the electron spin with the magnetic field arising from electron motion in hydrogen-like atom is given by

$$U_{SO} = \frac{g_s}{4m^2c^2r^2} \frac{dV}{dr} \vec{L} \cdot \vec{S} = -\frac{kZe^2}{2m^2c^2r^3} \vec{L} \cdot \vec{S}, \quad (1)$$

where $V = \frac{kZe^2}{r}$, Z is the atomic number, k is the Coulomb constant, m is the electron mass, c is the speed of light, r is the radial distance of the electron from the nucleus, and $g_s = 2$ is the gyromagnetic ratio.

In gravitomagnetism theory, we have shown that [1],

$$U_{SO-gm} = \frac{\pi g'_s GM^2}{4m^2c^2r^3} \vec{L} \cdot \vec{S}, \quad (2)$$

g'_s is the gravitational gyromagnetic ratio, will correspond to an interaction gravitomagnetic energy

$$U_{gm} = -\frac{\pi}{3} \frac{GML^2}{2mc^2r^3}, \quad (3)$$

However, Einstein's theory of gravitation employing Schwartzchild metric shows that because of space curvature a term of

$$U_{GR} = -\frac{GML^2}{2mc^2r^3}, \quad (4)$$

appears in the total energy of the gravitating object.

Thus, eq.(3) and (4) very close to each other. This minute difference between the two paradigm should be further explored. Notice that the inclusion of energy momentum in Einstein relativity equations leads to the space curvature, whereas the inclusion of spin would lead to the space torsion. Einstein's general relativity respects the former but not the latter case. While Einstein interprets the precession of planets to curvature of space, we ascribe it to the interaction of the spin of planets with gravitomagnetic field of the Sun. This interpretation complies with atomic description. Consequently, the gravitomagnetism would suggest a curved space due to the presence of electric charge that is tantamount to presence of (mass) matter that is known to curve space.

Assuming the spin-orbit coupling as responsible for precession of perihelion of planetary orbits, the spin of a planet of mass m orbiting a star of mass M' can be obtained by equating eqs.(2) and (3), *i.e.*, spin-orbit interaction energy equals to gravitomagnetic energy, which yields

$$S = \frac{\alpha}{\cos \theta} \frac{m}{M} L, \quad (5)$$

where L is the orbital angular momentum of the orbiting planet, $\alpha = \frac{2g'_s}{3}$, θ is the angle between L and S directions, e is the orbit eccentricity, and $M = (M' + m)^{\dagger 4}$ is the total

⁴ $M \sim M'$

mass of the system. However, the orbital plane of most planets is inclined to their ecliptic with an angle (i), so that in this case eq.(5) becomes

$$S = \alpha \frac{\cos i}{\cos \theta} \frac{m}{M} L, \quad (6)$$

Equation (5) suggests that for a system of particle each has a mass of m_i and an angular momentum of L_i , one has a center of mass angular momentum

$$S \approx \frac{\sum_i m_i L_i}{\sum_i m_i}, \quad (7)$$

so that the spin represents the center of mass of the angular momenta of the system.

Owing to the apparent analogy between electromagnetism and gravitomagnetism, one has ⁵

$$G \rightarrow k, \quad GMm \rightarrow kZe^2, \quad (8)$$

so that eq.(1) reads

$$U'_{SO} = \frac{g_g GM}{4mc^2 r^3} \vec{L} \cdot \vec{S}, \quad (9)$$

where g_g is gravitational gyromagnetic ratio analogue. It is not known whether $g_g = 2$ or not. Hence, equating eqs.(4) and (8) implies that $S = \frac{L}{\cos \theta}$ for $g_g = 2$. This is however not correct for the planetary system. Equation (8) agrees with eq.(2) only if $g_g = \pi \frac{M}{m} g'_s$.

We remark that several authors have considered the gravitational spin-orbit coupling comparing it with the atomic analogue [2, 3]. None of them have derived it from first principle, or equivalently didn't show how the gravitational spin magnetic moment is related to the spin. This is only done in our recent publication [1]. Mashhoon proposed that the analogy between gravity and electromagnetism dictates that charge, $q \rightarrow -2m$. Hence, he concluded that the gravitational magnetic moment due to spin is related to spin by $\mu_s = -S$ [4]. In our gravitomagnetic theory, this is however related by the relation $\mu_g = \frac{M}{2m} S$. If we write eq.(5) as $S = \beta L$ and the total angular momentum $\vec{J} = \vec{L} + \vec{S}$, then $\vec{L} \cdot \vec{S} = \beta L^2$. Applying eq.(7) in eq.(3) dictates that the curvature term lead to a potential (interaction) energy in the atomic system

$$U_{em} = -\frac{\pi}{3} \frac{kZe^2 L^2}{2m^2 c^2 r^3}. \quad (10)$$

Therefore, one can write the total potential energy for an electron in hydrogen-like atoms in an electrically space as

$$E = -\frac{kZe^2}{r} + \frac{L^2}{2mr^2} - \frac{\pi}{3} \frac{kZe^2 L^2}{2m^2 c^2 r^3}. \quad (11)$$

Comparing eqs.(1) and (9) reveals that $\vec{L} \cdot \vec{S} = \frac{\pi}{3} L^2$ for an atomic system. Thus, with this understanding the space inside an atom is not flat space (Minkowskian), but follows

⁵For non-hydrogen-like atom $e^2 \rightarrow (Ze)e$, $e \rightarrow m$, and $Ze \rightarrow M$

Schwarzschild pattern, where $\frac{2GM}{c^2} \rightarrow \frac{2kZe^2}{mc^2}$ is the electrical Schwarzschild radius. Hence, the metric for a spherically symmetric distribution of nuclear matter can be written as

$$ds^2 = c^2 \left(1 - \frac{2kZe^2}{mc^2 r}\right) dt^2 - \frac{dr^2}{1 - \frac{2kZe^2}{mc^2 r}} - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (12)$$

One can then interpret the Rutherford- α deflection as a consequence of the electrical curvature inside the atom. This is tantamount to deflection of light by the Sun curvature. The electric Schwarzschild radius is equal to twice classical electron radius, $R_s = 2r_c = \frac{2kZe^2}{mc^2}$. Similarly, one would expect a photon to be electrically redshifted in an electrical potential of the nucleus by an amount, $z = \frac{kZe^2}{mc^2 r}$ in hydrogen-like atoms. Hence, in an analogous manner when light passes near a central charge it will experience a redshift. This can be written as, $z = \alpha Z \left(\frac{\lambda_C}{r}\right)$, where $\lambda_C = \frac{h}{mc}$ is Compton wavelength of the electron. Now, when $r = \lambda_C$, then $z_m = \alpha Z$. This case represents a maximal (quantum) redshift. Thus, owing to the one-to-one analogy between gravitation and electromagnetism, one can use Einstein's general relativity to describe electromagnetic phenomena, and Maxwell's equation to describe gravitational phenomena.

The planetary spin and radius. — The origin of spin of planets has not been known exactly. One can easily determine the orbital angular momentum of a planet. The spin of a planet however requires knowledge of the planet mass, radius, its rotation period and its mass distribution inside the planet. Since some planets are solid (rocky) and other are gaseous, it is not easy to identify precisely their internal structure. The former ones have generally higher rotation rate than the latter. However, orbital periods of planets depend on their distance from the Sun and the Sun mass only. We provide here a formula for spin or rotational period from its orbital motion only. Or equivalently, we relate the spin to the orbital angular momentum for the first time in history.

Equation (6) can be used to express the planetary spin as

$$S = \alpha \frac{\cos i G m^2}{v \cos \theta}. \quad (13)$$

This is a very interesting and useful formula that can be used to calculate the spin of a planet without resort to its rotation period and its radius. Delauney and Flammarion related the spin period of a planet to a host of planetary physical characteristics concluding that there is a direct relationship between spin period and mean density [10]. Furthermore, Brosche noticed that some planets in a similar size range had spin angular momenta, S , that were proportional to the squares of their masses, m [9, 10]

$$S \propto m^2. \quad (14)$$

Equation (13) agree partially with eq.(12). In rotational dynamics the spin of a rigid body

(object) planet is defined by

$$S = I\omega, \quad \omega = \frac{2\pi}{D}, \quad I = \lambda m R^2, \quad (15)$$

where λ is the coefficient of inertia, R the planet's radius, and D is the rotational period of the planet. Equation (12) and (14) states that the radius of the planet is

$$R = \left(\frac{\alpha \cos i}{2\pi \lambda \cos \theta} \frac{G m D}{v} \right)^{1/2}. \quad (16)$$

Using eqs. (6) and (14) one can write

$$\frac{D}{P} = \frac{1}{C^2} \left(\frac{R}{a} \right)^2 \frac{M}{m}, \quad C = \left(\frac{\alpha \cos i \sqrt{1-e^2}}{\lambda \cos \theta} \right)^{1/2}, \quad (17)$$

where P and a are, respectively, the orbital period and the semi-major axis of the orbiting planet. Equation (16) can be written as

$$D = \frac{1}{C^2} \left(\frac{4\pi^2}{GM} \right)^{2/3} \left(\frac{M}{m} \right) R^2 P^{-1/3}. \quad (18)$$

An educated guess can relate α to the ellipticity (flattening/oblateness) of the planet, or to the eccentricity of the orbit. Using eq.(14) the rotation spin rate can be written as

$$\omega = \alpha \frac{\cos i}{\cos \theta} \left(\frac{\sqrt{G M m^2 a(1-e^2)}}{\lambda M R^2} \right), \quad (19)$$

Equation (18) can be written as

$$\omega = \alpha \frac{\cos i}{\cos \theta} \frac{L}{I_\lambda}, \quad I_\lambda = \lambda M R^2, \quad L = \sqrt{G M m^2 a(1-e^2)}. \quad (20)$$

The radius of a planet that is tidally locked to its star, *i.e.*, $P = D$, is given by (*see eq.(16)*)

$$R_t = C \sqrt{\frac{m}{M}} a. \quad (21)$$

Equation (16) can also be written as, for $P = D$,

$$P_t^2 = C^{-3} \left(\frac{4\pi^2}{GM} \right) \left(\frac{M}{m} \right)^{3/2} R_t^3. \quad (22)$$

It is of prime interest to mention that a hypothetical satellite that had a circular orbit radius equals to the radius of the planet, R , its orbital period P is given by [10]

$$P_R^2 = \left(\frac{4\pi}{GM} \right) R^3. \quad (23)$$

Flammarion calculated P_R values for Earth, Jupiter, Saturn, Uranus and Neptune, by extrapolating Keplers Harmonic Law, as applied to their known satellites [10]. Only some of these period are in agreement with observation.

The radius of a black hole is related to its mass, m , by

$$R = \frac{2Gm}{c^2}. \quad (24)$$

Therefore, the gravitational force for such a planet (spinning black hole) is given by

$$F_N = \frac{G m M}{r^2} = C^2(1 - e^2) \frac{c^4}{4G} \frac{D}{P}, \quad (25)$$

where $r = a(1 - e^2)$. This clearly shows that a spinning black hole will experience a huge gravitational force when orbits any central massive object.

This force is maximum when the planet is tidally-locked to its star, *i.e.*, $P = D$. Hence, one has

$$F_N^{max.} = C^2(1 - e^2) \frac{c^4}{4G}. \quad (26)$$

This force is of the order of $\frac{c^4}{G}$. It is however shown that the maximal force in nature is defined by $F_{max.} = \frac{c^4}{4G}$ [5, 6]. It also represents the maximum self-gravitating mass. It is thus interesting to see that the gravitational force arising from this case is of the same order of this maximal force. For a black hole planet of radius R_p tidally-locked with a black hole star with radius R_s , one has

$$R_p R_s = C^2 a^2. \quad (27)$$

This is an interesting relation connecting the two radii of orbiting black that are tidally-locked to their semi major axis. Moreover, it is clear that the existence of such a system awaits the future astronomical exploration.

Binary pulsars. – A pulsar is a rotating neutron star that emits intense beam of electromagnetic radiation. A binary pulsar consist of a pulsar orbiting a neutron star or white dwarf (companion). The very famous binary pulsar is the PSR B1913+16 discovered by Hulse and Taylor [8] The orbital and rotation periods are, respectively, 7.75 hours and 0.059 sec.

It is apparent from eq.(1) that for binary pulsars where $m \sim M$, $L \sim S$. Hence, eq.(16) reveals that

$$R \sim \sqrt{\frac{D}{P}} a. \quad (28)$$

The pulsar data can be used to validate eq.(17) and to determine the constant α . We have recently place a limit on neutron star higher than those postulated by that of the standard one [11]. Accordingly, the radius of neutron star is given by

$$R_N = \left(\frac{3k^3 e^6 G}{5m_e^2 c^8} \right)^{1/4} \left(\frac{M}{m_e} \right)^{1/2} = 80.9 \left(\frac{M}{M_\odot} \right)^{1/2} (\text{km}), \quad (29)$$

where m_e is the mass of the electron and M_\odot is the solar mass. The radii below in Table 1 are calculated from eq.(29). We consider here the planetay system, Jupiter satellites, and

Saturn satellites. The constant C is calculated using eq.(18) and Tabulated in Tables 1, 2, 3, 4, 5 and 6 [12]. The average value of C are 0.055, 0.089, 0.077, and 0.068, for binary pulsars, Jupiter satellites, Saturn satellites, and planetary system, respectively. Notice that for exoplanet and asteroids the constant C takes the average values 7 and 0.5, respectively. The higher of C for asteroids may be attributed to the uncertainty associated with the observational data related to them. Table 6 can be used to identify exoplanets that are tidally locked by comparing the resulting C for a given system with that of the Moon. Since our Moon is tidally locked and well-known, we can consider $C = 0.041$ for tidally locked planets. With this value we complete table 7. Assuming the exoplanetary system is similar to our planetary system, we suggest that $C = 0.1$, as evident from table 2. With this value we calculate the day for some exoplanets as shown in table 8. Notice that Mercury is very closed to tidally-locked system envisaged in Tables 3 and 7.

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<http://nssdc.gsfc.nasa.gov/planetary/factsheet/>, and <http://exoplanet.eu>

| Name | $m (M_{\odot})$ | $M (M_{\odot})$ | $P (day)$ | $D (s)$ | e | $R (km)$ | C |
|-----------------|-----------------|-----------------|-----------|---------|-----------|----------|--------|
| PSR B1913+16 | 1.441 | 2.828 | 0.323 | 0.05903 | 0.617131 | 97.22 | 0.060 |
| PSR B1534+12 | 1.333 | 2.678 | 0.4207 | 0.03790 | 0.2736767 | 93.48 | 0.072 |
| PSR B2127+11C | 1.350 | 2.713 | 0.3353 | 0.0305 | 0.681 | 94.08 | 0.083 |
| PSR J0737-3039A | 1.34 | 2.58 | 0.102 | 0.0227 | 0.0878 | 93.73 | 0.116 |
| PSR J1518+4904 | 1.75 | 2.62 | 8.634 | 0.04093 | 0.2494 | 107.02 | 0.0414 |
| PSR B2303+46 | 1.16 | 2.53 | 12.340 | 1.06637 | 0.658369 | 87.21 | 0.0076 |

Table 1: Binary pulsars primary data

| Name | $m(\times 10^{24} \text{kg})$ | $P (day)$ | $D (hr)$ | e | $a (au)$ | $R (km)$ | C |
|---------|-------------------------------|-----------|----------|----------|----------|----------|-------------|
| Mercury | 0.3302 | 87.969 | 1407.6 | 0.2056 | 57.91 | 2439.7 | 0.036390953 |
| Venus | 4.8685 | 224.701 | 5832.5 | 0.0067 | 108.21 | 6051.8 | 0.009877991 |
| Earth | 5.9736 | 365.256 | 23.9345 | 0.0167 | 149.6 | 6378.1 | 0.13530133 |
| Mars | 0.64185 | 686.98 | 24.6229 | 0.0935 | 227.92 | 3396.2 | 0.195060979 |
| Jupiter | 1898.60 | 4332.59 | 9.925 | 0.04093 | 778.57 | 71492 | 0.087422946 |
| Saturn | 568.46 | 10759.22 | 10.656 | 0.0489 | 1433.53 | 60268 | 0.111249286 |
| Uranus | 86.832 | 30685.40 | 17.24 | 0.0565 | 2872.46 | 25559 | 0.079987318 |
| Neptune | 102.43 | 60189 | 16.11 | 0.0457 | 24764 | 4495.06 | 0.066062714 |
| Pluto | 0.01305 | 89866 | 153 | 0.244671 | 5874 | 5874 | 0.082682797 |

Table 2: The planetary system primary data

| Name | $m(\times 10^{22} \text{kg})$ | $a (km)$ | $P (day)$ | $D (day)$ | e | $R (km)$ | C |
|------|-------------------------------|----------|-----------|-----------|--------|----------|-------|
| Moon | 7.3477 | 384400 | 27.321582 | 27.321582 | 0.0549 | 1738.14 | 0.041 |

Table 3: The Moon (tidally locked to the Earth) primary data

| Name | $m(\times 10^{20} \text{kg})$ | $a (10^3 \text{km})$ | $P (day)$ | $D (day)$ | e | $R (km)$ | C |
|----------|-------------------------------|----------------------|-----------|-----------|--------|----------|--------|
| Io | 893.2 | 421.6 | 1.769138 | 1.769138 | 0.004 | 1821.6 | 0.0116 |
| Europa | 480 | 670.9 | 3.551181 | 3.551181 | 0.0101 | 1560.8 | 0.0085 |
| Ganymede | 1481.9 | 1070.4 | 7.1545535 | 7.154553 | 0.0015 | 2631.2 | 0.0051 |
| Callisto | 1075.9 | 1882.7 | 16.689018 | 16.689018 | 0.007 | 2410.3 | 0.0031 |
| Elara | 0.008 | 11740 | 259.6528 | 0.5 | 0.217 | 40 | 0.0695 |
| Himalia | 0.095 | 11460 | 250.5662 | 0.4 | 0.162 | 85 | 0.0483 |
| Metis | 0.001 | 128 | 0.294779 | 0.294779 | 0.0002 | 20 | 0.3959 |

Table 4: Jupiter Satellites primary data

| Name | $m(\times 10^{20} \text{kg})$ | $a (10^3 \text{km})$ | $P (day)$ | $D (day)$ | e | $R (km)$ | C |
|---------|-------------------------------|----------------------|-----------|-----------|--------|----------|--------|
| Miranda | 0.66 | 129.39 | 1.413479 | 1.413479 | 0.0027 | 235.8 | 0.1797 |
| Ariel | 13.5 | 191.02 | 2.520379 | 2.520379 | 0.0034 | 578.9 | 0.0661 |
| Umbriel | 11.7 | 266.3 | 4.144177 | 4.144177 | 0.005 | 584.7 | 0.0514 |
| Titania | 35.2 | 435.91 | 8.705872 | 8.705872 | 0.0022 | 788.9 | 0.0244 |
| Oberon | 30.1 | 583.52 | 13.463239 | 13.463239 | 0.0008 | 761.4 | 0.0191 |

Table 5: Saturn Satellites (tidally locked) primary data

| Name | $m(\times 10^{19}\text{kg})$ | a (au) | P (year) | D (day) | e | R (km) | C |
|---------|------------------------------|----------|------------|-----------|---------|----------|-------|
| Ceres | 87 | 2.767 | 4.6 | 9.075 | 0.0789 | 487.3 | 3.75 |
| Juno | 2 | 2.669 | 4.36 | 7.21 | 0.2579 | 120 | 6.90 |
| Vesta | 30 | 2.362 | 3.63 | 5.342 | 0.0895 | 265 | 4.71 |
| Eugenia | 0.61 | 2.721 | 4.49 | 5.699 | 0.0831 | 113 | 13.16 |
| Siwa | 0.15 | 2.734 | 4.51 | 18.5 | 0.2157 | 51.5 | 6.70 |
| Chiron | 0.4 | 13.633 | 50 | 5.9 | 0.3801 | 90 | 8.48 |
| Haumea | 41.79 | 43.335 | 285.4 | 3.912 | 0.18874 | 718 | 6.11 |
| Pallas | 3.18 | 2.7707 | 3.62 | 7.8132 | 0.231 | 261 | 10.03 |
| Eris | 1.62 | 2.385 | 3.68 | 7.14 | 0.231 | 199.8 | 13.18 |

Table 6: Asteroids primary data

| Name | $m(M_J)$ | $M(M_\odot)$ | a (au) | $R(R_J)$ | P (day) | C |
|-----------------|----------|--------------|----------|----------|-----------|--------|
| Kepler-34(AB) b | 0.22 | 2.0687 | 1.0896 | 0.76 | 288.822 | 0.0350 |
| Kepler-9 c | 0.171 | 1 | 0.225 | 0.823 | 38.9086 | 0.0395 |
| KOI-55 c | 0.0021 | 0.496 | 0.0076 | 0.078 | 0.34289 | 0.0395 |
| HD 97658 b | 0.02 | 0.85 | 0.0797 | 0.262 | 9.4957 | 0.0329 |
| GJ 3470 b | 0.044 | 0.541 | 0.0348 | 0.376 | 3.33714 | 0.0453 |
| Kepler-22b | 0.11 | 0.97 | 0.85 | 0.214 | 289.9 | 0.0425 |
| Gl 581 g | 0.01 | 0.31 | 0.14601 | 0.0678 | 36.652 | 0.0429 |

Table 7: Tidally-locked Exoplanets primary data

| Name | $m(M_J)$ | $M(M_\odot)$ | a (au) | P (day) | $R(R_J)$ | D (day) |
|---------------|----------|--------------|----------|-----------|----------|-----------|
| WASP-10 b | 3.06 | 0.71 | 0.0371 | 3.09276 | 1.08 | 0.6566 |
| XO-5 b | 1.077 | 0.88 | 0.0487 | 4.18775 | 1.03 | 0.4783 |
| WASP-16 b | 0.855 | 1.022 | 0.042 | 3.1186 | 1.008 | 0.1891 |
| KOI-204 b | 1.02 | 1.19 | 0.0455 | 3.24674 | 1.24 | 0.1564 |
| XO-2 b | 0.62 | 0.98 | 0.0369 | 2.61584 | 0.973 | 0.0993 |
| TrES-1 | 0.761 | 0.88 | 0.0393 | 3.03007 | 1.099 | 0.1398 |
| WASP-1 b | 0.86 | 1.24 | 0.0382 | 2.51995 | 1.484 | 0.0483 |
| HAT-P-17 b | 0.534 | 0.857 | 0.0882 | 10.3385 | 1.01 | 2.051 |
| WASP-55 b | 0.57 | 1.01 | 0.0533 | 4.46563 | 1.3 | 0.1768 |
| WASP-6 b | 0.503 | 0.888 | 0.0421 | 3.36101 | 1.224 | 0.0940 |
| 55 Cnc e | 0.0263 | 0.905 | 0.0156 | 0.736546 | 0.194 | 0.00578 |
| OGLE2-TR-L9 b | 4.34 | 1.52 | 0.0308 | 2.48553 | 1.614 | 0.1079 |
| OGLE-TR-10 b | 0.68 | 1.18 | 0.04162 | 3.10129 | 1.72 | 0.04368 |
| XO-3 b | 11.79 | 1.213 | 0.0454 | 3.19152 | 1.217 | 1.802 |
| PSR 1719-14 b | 1 | 1.4 | 0.0044 | 0.0907063 | 0.4 | 0.000327 |
| WASP-14 b | 7.341 | 1.211 | 0.036 | 2.24377 | 1.281 | 0.4484 |
| HD 80606 b | 3.94 | 0.98 | 0.449 | 111.436 | 0.921 | 4444.9 |

Table 8: Some non-tidally locked Exoplanets primary data. The day is obtained for $C = 0.1$ as can be guessed from table 2 for planetary system.